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Repetitive Discrete Processes Based Iterative Learning Control designed by LMIs

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***Abstract*—This paper addressed the stability analysis along the pass and the synthesis problem of linear 2D/repetitive systems. The proposed conditions guarantee of the system, the stability in the closed-loop. The given results are expressed in terms of linear matrix inequality (LMI). Simulation results demonstrate the good performance of the theoretical scheme.**

***Keywords*—Systems Repetitive, iterative learning control (ILC), 2D/repetitive systems, Stability along the pass, LMIs.**

I. Introduction

Repetitive processes are a distinct class of two-dimensional 2-D linear systems (i.e. information propagation in two independent directions) of widely spread over industrial fields. The essential unique characteristic of such process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length [1, 2, 3, 4]. On each pass, an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction. To introduce a formal definition, let $\alpha < +\infty$ denotes the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(p), 0 \le p \le \alpha,\ k \ge 0$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(p),\ 0 \le p \le \alpha - 1,\ k \ge 0$. The fact that the pass length is finite (and hence information in this direction only occurs over a finite duration) is the key difference with other classes of 2D systems, such as those with discrete dynamics described by the well known and extensively studied Roesser and Fornasini Marchesini state space models [2, 3]. Physical examples of repetitive processes, considering a robot that has to undertake a picking and placing manipulation.

Once the task is achieved, the robot is reset to the initial position and then the task is repeated. Also in recent years, applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications include classes of iterative learning control (ILC) schemes [5, 6, 7].

Iterative learning control (ILC) systems have gained much attention during the last decade, which deserve investigation for theoretical development as well as for practical applications [7, 8]. ILC is a technique especially developed for repetitive process, which requires repeating the same operation or task, over a finite duration and constant $\alpha \in [0,T]$. The objective of ILC is to make the output $y_k(p)$, produced on the k^{th} pass acts as a forcing function on the next pass and hence contributes to the dynamics of the new pass profile $y_{k+1}(p), 0 \le p \le \alpha - 1, k \ge 0.$ The original work in this area (ILC) is created by [1]. In [5], determine the conditions under which error convergence trial-to-trial, it is possible to convergence pass to pass to a limit error which has unacceptable along the trial dynamic.

This note is organized as follows. In Section 2, we introduce the theoretical study of stability along the pass of a discrete linear repetitive process. Section 3, applying the Iterative Learning Control for discrete SISO system, and a performance analysis of ILC systems by mean of a quadratic Lyapunov function is investigated. In Section 4, a new sufficient LMI condition is demonstrated, to obtain stabilizing classes of 2D systems. Then, a numerical evaluation is presented to illustrate the effectiveness of the proposed approach in Section 5. Finally, the paper is concluded in Section 6.

Throughout this paper, X > 0(respectively, < 0) denotes a real symmetric positive (respectively, negative) definite matrix. A^T denotes the transpose of A. To simplify the scriptures, we will use the symbol $sym\{A\} = A^T + A.*$ is used for the blocks induced by symmetry. Also the identity and null matrix of the required dimensions are denotes by I and 0, respectively.

II. Stability theory of a discrete linear repetitive process

In this section, we discuss the concept of a repetitive control system, and recall the main stability theorem for such systems. The state space model, of a discrete linear repetitive process [7, 8], described by the following form over $0 \le p \le \alpha - 1,\ k \ge 0$.

$$\begin{cases} x_{k+1}(p+1) = Ax_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p), \\ y_{k+1}(p) = Cx_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p). \end{cases} \quad (1)$$

Here on pass k , $\alpha < +\infty$: denotes the pass length (α : is the finite pass length), $x_k(p) \in \mathbb{R}^n$:is the state vector, $u_k(p) \in \mathbb{R}^r$: is the input vector, $y_k(p) \in \mathbb{R}^r$:is the output or pass profile.

Then, stability along the pass holds if, and only if, the so-called 2D characteristic polynomial [6, 8]:

$$C_{disLRP} = det\left(\begin{bmatrix} I - z_1 A & -z_1 B_0 \\ -z_2 C & I - z_2 D_0 \end{bmatrix}\right) \neq 0. \quad (2)$$

Where $z_1, z_2 \in \mathbb{C}$, consist of two distinct operators in the along the pass (p) and pass to pass (k) directions respectively as

$$\begin{cases} x_k(p) = z_1 x_k(p+1), \\ x_k(p) = z_2 x_{k+1}(p). \end{cases} \quad (3)$$

Theorem 1: [9] a discrete linear repetitive process of the form (1) (controllable and observable) is stable along the pass if and only if,

(1) $\rho(D_0) < 1,$

(2) $\rho(A) < 1,$

(3) $\left|G_{dis}(z^{-1})\right| = \left|C\left(z_1^{-1} I - A\right)^{-1} B_0 + D_0\right| < 1, \forall |z| = 1,$ all eigenvalues of $G_{dis}(z^{-1})$ have modulus strictly less than one.

All three conditions of the Theorem 1 have well-defined physical interpretations and, unlike equivalents [10], can be tested by direct application of 1D linear time invariant systems. It is easy to show that stability along the pass guarantees that the corresponding limit profile of (1) is stable as a 1D linear system, i.e. all eigenvalues of the state matrix (setting $D = 0$ for simplicity) $A + B_0 (I - D_0)^{-1} C$ have strictly negative real parts.

In terms of checking the conditions of these two results, the first two conditions in each case are easily solves.

$\rho(D_0) < 1$, this is the necessary and sufficient condition for asymptotic stability, i.e. BIBO stability over the finite pass length.

Applying the second conditions of Theorem 1, stability of the matrix A (i.e. a uniformly bounded first pass profile) is, in general, only a necessary condition for stability along the pass.

The only difficulty, which can be arising, is the computational cost associated with, condition (3). For SISO examples, this condition requires that the Nyquist plot generated by $G_{dis}(z^{-1})$ lies inside the unit circle in the complex plane for all $|z^{-1}| = 1.$

III. APPLICATION TO ITERATIVE LEARNING CONTROL(ILC)

In this section, the subject is use control law design for the system LTI. We considered a discrete linear time invariant system described by the state space $\{A, B, C\}$ is considered:

$$\begin{cases} x_k(p+1) = Ax_k(p) + B u_k(p), \ 0 \le p \le \alpha - 1 \\ y_k(p) = Cx_k(p). \end{cases} \quad (4)$$

Where, on trial k the signal to be tracked is denoted by $y_d(p)$ then $e_k(p) = y_d(p) - y_k(p)$, is the error on trial k .

Let a control law given by:

$$u_{k+1}(p) = u_k(p) + K_1 \eta_{k+1}(p+1) + K_2 e_k(p+1). \quad (5)$$

Then clearly (4) and (5) can be written as:

$$\begin{cases} \eta_{k+1}(p+1) = x_{k+1}(p) - x_k(p) \\ \qquad = \tilde{A}\eta_{k+1}(p) + \tilde{B}_0 e_k(p), & (6) \\ e_{k+1}(p) = y_d(p) - y_{k+1}(p) \\ \qquad = \tilde{C}\,\eta_{k+1}(p) + \tilde{D}_0 e_k(p). & (7) \end{cases}$$

We also introduce these variables:

$$\begin{cases} \tilde{A} = A + BK_1, \\ \tilde{B}_0 = BK_2, \\ \tilde{C} = -C(A + BK_1), \\ \tilde{D}_0 = I - CBK_2. \end{cases} \quad (8)$$

Then, clearly (6) and (7) can be written as:

$$\begin{bmatrix} \eta_{k+1}(p+1) \\ e_{k+1}(p) \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{B}_0 \\ \tilde{C} & \tilde{D}_0 \end{bmatrix} \begin{bmatrix} \eta_{k+1}(p) \\ e_k(p) \end{bmatrix}. \quad (9)$$

It has been proved recently that any robust control problem can be turned into an LMI dilated one, in terms of converting the Lyapunov conditions to be generalized in equations by mean of lemmas [8, 9].

However, it is very difficult to provide computationally effective tests for stability in this way.

One of the ways to derive tractable tests is by applying Lyapunov theory associated with LMI techniques that became a standard tool for the stability analysis of 1-D system when manipulating state space models.

These Lyapunov functions must contain contributions from the current pass state and previous pass profile vectors, for example, composed of which is the sum of quadratic terms in the current pass state and previous pass profile respectively [6, 8].

This approach is developed by using candidate Lyapunov function for discrete models, having the following form:

$$V\left(k,p\right)=x_{k+1}^{T}\left(p\right)P_1x_{k+1}\left(p\right)+y_{k}^{T}\left(p\right)P_2y_{k}\left(p\right). \qquad (10)$$

Where, $P_1>0$ and $P_2>0$.

With associated increment:

$$\Delta V\left(k,p\right)=x_{k+1}^{T}\left(p+1\right)P_1x_{k+1}\left(p+1\right)-x_{k+1}^{T}\left(p\right)P_1x_{k+1}\left(p\right) + y_{k+1}^{T}\left(p\right)P_2y_{k+1}\left(p\right)-y_{k}^{T}\left(p\right)P_2y_{k}\left(p\right).$$

Then the stability along the pass holds if $\Delta V\left(k,p\right)<0$ for all k and p which is equivalent to the requirement that:

$$\Phi_i^T P_{i+1}\Phi_i - P_i < 0. \qquad (11)$$

Where: $P_i = diag\left(P_1,P_2\right)$ and $\Phi>0.$

IV. LMI BASED ITERATIVE LEARNING CONTROL

In this section, the Kalman-Yakubovich-Popov (KYP) lemma [10, 11], is used as a basis idea to develop necessary and sufficient conditions for stability along the pass of the SISO of the discrete linear repetitive processes (4).

The KYP lemma is expressed as follows,

Lemma: [10, 11] for a given transfer

Function $G(z)=\tilde{C}\left(zI-\tilde{A}\right)^{-1}\tilde{B}_0+\tilde{D}_0$, the following inequality: $\begin{bmatrix}G(z) & I\end{bmatrix}\Pi\begin{bmatrix}G^*(z)\\ I\end{bmatrix}<0.$

Holds if, and only if, the exits Hermitian matrices $P>0$ and $Q>0$ such that

$$\begin{bmatrix}\Gamma(P,Q)+\Lambda & \begin{bmatrix}\tilde{B}_0\\ \tilde{D}_0\end{bmatrix}\\ \begin{bmatrix}\tilde{B}_0 & \tilde{D}_0\end{bmatrix} & -1\end{bmatrix}<0,$$

where

$$\Gamma\left(P,Q\right)+\Lambda=\begin{bmatrix}\tilde{A} & I\\ \tilde{C} & 0\end{bmatrix}\begin{bmatrix}P & -Q\\ -Q & -P+2Q\end{bmatrix}\begin{bmatrix}\tilde{A} & \tilde{C}\\ I & 0\end{bmatrix}^T+\begin{bmatrix}0 & 0\\ 0 & -1\end{bmatrix}.$$

Theorem 2: [7] the SISO version of (9) is stable along the pass if and only if there exist matrices $r>0, S>0, Q>0$ and a symmetric matrix P such that the following LMIs are feasible

$$(1)\ \tilde{D}_0^T r\tilde{D}_0 - r<0,$$

$$(2)\ \tilde{A}^T S\tilde{A}-S<0, \qquad (12)$$

$$(3)\begin{bmatrix}\tilde{A}P\tilde{A}^T-P-Q\tilde{A}^T-\tilde{A}Q+2Q & \tilde{A}P\tilde{C}^T-Q\tilde{C}^T & \tilde{B}_0\\ \tilde{C}P\tilde{A}^T-\tilde{C}Q & \tilde{C}P\tilde{C}^T-I & \tilde{D}_0\\ \tilde{B}_0^T & \tilde{D}_0^T & -I\end{bmatrix}<0.$$

The difficulty with the condition of Theorem 2 is that it is non-linear in its parameters. It can, however, be controlled in to the following results, where the inequality is a strict LMI a linear constraint which also gives a formula for computing K.

Theorem 3: The SISO version of (9) is stable along the pass if there exist matrices $S>0, Q>0, G, N_g, K_2$ and a symmetric matrix P such that the following LMIs are feasible:

$$(1)\begin{bmatrix}-CBK_2 & 0\\ 0 & CBK_2-2\end{bmatrix}<0, \qquad (13)$$

$$(2)\begin{bmatrix}-S & AG+BN_g\\ * & S-G-G^T\end{bmatrix}<0, \qquad (14)$$

$$(3)\begin{bmatrix}-P+2Q+sym\left\{\alpha AG+\alpha BN_g\right\} & * & -Q-\alpha G^T+AG+BN_g & BK_2\\ -\alpha CAG-\alpha CBN_g & -I & -CAG-CBN_g & I-CBK_2\\ * & * & P-G-G^T & 0\\ * & * & 0 & -I\end{bmatrix}<0. \qquad (15)$$

Where $\alpha>0$. If these LMIs are feasible, the controller gain is computed by: $K_1=N_gG^{-1}$.

Proofs

1- First LMI: First note that both r and, $\tilde{D}_0$ are real numbers and hence $r\left(\tilde{D}_0^2-1\right)<0$, with $r>0$. By, using (8), it is obvious that $\left(1-CBK_2\right)^2-1<0$, or $CBK_2\left(CBK_2-2\right)<0.$

Hence, we require $0<CBK_2<2$. The value of CBK_2 greatly influences the pass to pass error convergence, which is equivalent to (13) since here CBK_2 is a scalar.

2- Second LMI : $\tilde{A}^T S\tilde{A}-S<0$, by applying the schur complement, this inequality is equivalent in the first step to :

$$\begin{bmatrix} -S & S\tilde{A}^T \\ * & -S \end{bmatrix} < 0, \qquad (16.1)$$

and by using the projection lemma [12], we obtain:

$$(i)\begin{bmatrix} -S & 0 \\ 0 & -S \end{bmatrix} + \begin{bmatrix} S \\ 0 \end{bmatrix}\begin{bmatrix} 0 & \tilde{A}^T \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{A} \end{bmatrix}\begin{bmatrix} S & 0 \end{bmatrix} < 0, \quad (16.2)$$

$$(ii)\begin{bmatrix} -S & 0 & S \\ * & -S & \tilde{A}G \\ * & * & -G-G^T \end{bmatrix} < 0, \qquad (16.3)$$

By applying the Schur complement, the inequality (16.3), is transformed into:

$$\begin{bmatrix} -S & \tilde{A}G \\ * & S-G-G^T \end{bmatrix} < 0.$$

Substituting (8) in this LMI, (14) is done.

3- Third LMI: Multiplied by $\begin{bmatrix} 1 & 0 & \tilde{A} \\ 0 & 1 & \tilde{C} \end{bmatrix}$, the right side of (15) and the left by its transpose.

Introducing (8), and by applying the projection lemma [12] the inequality (12) is obtained.

Moreover, (15) follows on setting $N_g = K_1 G$.

V. SIMULATION RESULTS

A. Illustrative example

We consider a discrete linear time invariant systems described by the state space $\{A, B, C\}$:

$$\begin{cases} x_k(p+1) = Ax_k(p) + Bu_k(p), \;\; 0 \le p \le \alpha - 1 \\ y_k(p) = Cx_k(p). \end{cases}$$

Where

$$A = \begin{bmatrix} -1.5 & 0.5 \\ 0.2 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix} \text{ and } C = \begin{bmatrix} 0.2 & 0.6 \end{bmatrix},$$

By applying the control law (5), the system is stable in the closed-loop and the control gains are computed:

$$K_1 = \begin{bmatrix} 1.483 & -3.684e^{-01} \end{bmatrix} \text{ and } K_2 = 1.460.$$

Consequently,

$$\rho(\tilde{A}) = \begin{Bmatrix} 1.460 \;\; e^{-0.1} \\ -2.989 \;\; e^{-0.1} \end{Bmatrix} < 1 \text{ and } \rho(\tilde{D}_0) = \{6.202e^{-0.1}\} < 1.$$

The next figure confirm the condition of Theorem 3

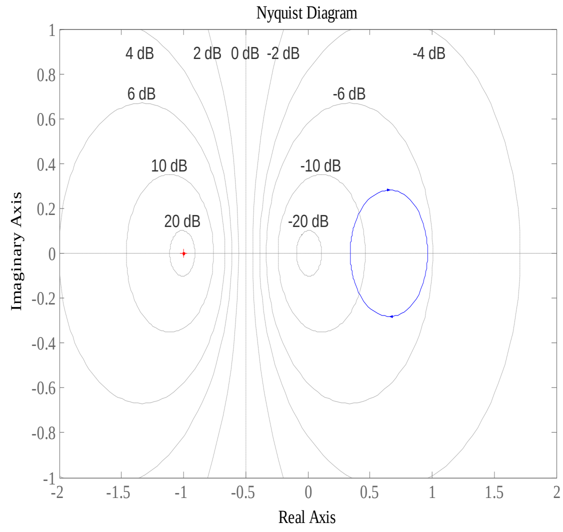

Fig. 1. Nyquist Diagram

B. Numerical Evaluation

In this section, we compare the control performances of two ILC algorithms (Theorem 3 [13] and Theorem 3 of the proposed approach) described above through a numerical evaluation summarized in the Table 1.

The system is characterized by order (n) and number of inputs (m). For fixed values of (n, m), we generate randomly 100 ILC systems of the form (4).

Method1: uses the conditions given in Theorem 3 [13], which are sufficient conditions.

Method2: uses the conditions given in Theorem 3 proposed in section 4, which are sufficient conditions.

By using the Matlab LMI Control Toolbox to check the feasibility of the LMI conditions, a counter is increased if the corresponding method succeeds in providing stabilizing control.

TABLE I. NUMERICAL EVALUATION

	Method	**success**
n=2 m=1	Method1 [13] Method2	72 86
n=3 m=1	Method1 [13] Method2	43 68
n=4 m=1	Method1 [13] Method2	24 40
n=5 m=1	Method1 [13] Method2	05 15
n=6 m=1	Method1 [13] Method2	01 06

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In this paper, the design of ILC schemes using a discrete linear repetitive processes setting is considered. Our main contribution consists in providing a new sufficient LMI condition for the control method for repetitive systems. A numerical evaluation is presented to illustrate the effectiveness of the proposed approach. The various conditions are given through a family of LMIs parameterized by scalar variable which offers an additional degree of freedom, enable at the expense of a relatively small degree of complexity in the numerical treatment (on line search) to provide better results compared to previous one.

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